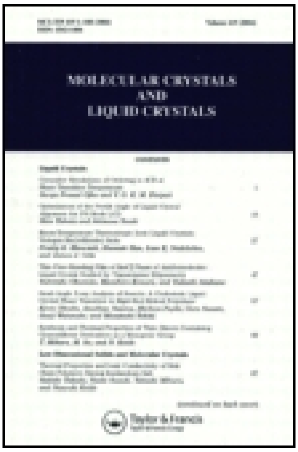

Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl19

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N. B. Baranova [a] , I. V. Goosev [a] , V. A. Krivoschekov [a] & B. Ya. Zel'Dovich [a]

[a] Nonlinear Optics Laboratory, Polytechnic Institute, Lenin prosp. 76, Chelyabinsk, 454080, USSR

Version of record first published: 24 Sep 2006.

Mol. Cryst. Liq. Cryst., 1992, Vol. 210, pp. 155–164
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Printed in the United States of America

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N. B. BARANOVA, I. V. GOOSEV, V. A. KRIVOSCHEKOV and B. YA. ZEL'DOVICH
Nonlinear Optics Laboratory, Polytechnic Institute, Lenin prosp. 76, Chelyabinsk, 454080, USSR

(Received June 19, 1991)

We predicted and confirmed experimentally that an ordinary wave may propagate through a very thick (~5 mm) nematic liquid crystal (NLC) cell almost without distortions. The physical idea is that: 1) the polarizations of the *e*- and *o*-wave follow adiabatically the local orientation of optical axis and 2) phase velocity of *o*-wave does not depend on orientation. Behavior of polarization around disclination line is studied.

1. INTRODUCTION

The most of liquid crystal (LC) displays use the property of ordinary or extraordinary wave to propagate with the polarization adiabatically following the local orientation of the director in a twisted NLC cell. At the same time, NLC in large (>1 mm) vessels are known to be very opaque. That opaqueness is connected with two reasons.

The first of them is rather high rate of scattering by thermodynamically equilibrium small-scale fluctuations of director, so that extinction coefficient $R \approx (1 \div 10)\ \text{cm}^{-1}$. That scattering could be slightly suppressed by external **E** or **H**-fields; however, the necessary values of E or H are unpractically large.

The second reason for a NLC cell not to transmit a good optical image through itself is that there are usually large-scale inhomogeneities of director, disclinations including, in a thick cell, $L > 1$ mm. The refraction of a wave by these smooth inhomogeneities is generally assumed as the source of the distortions of that part of image which was not scattered yet to relatively large angle by thermal fluctuations. It is so indeed for the extraordinary wave since the refractive index $n_e(\vartheta)$ strongly depends on the angle ϑ between the director $\mathbf{n}$ and propagation wave vector $\mathbf{k}$:

$$\vartheta = \vartheta(\mathbf{r}) = \arccos((\mathbf{kn})/k)$$

$$n_e(\vartheta) = n_\perp n_\parallel/(n_\perp^2 \sin^2\vartheta + n_\parallel^2 \cos^2\vartheta)^{1/2} \tag{1}$$

For $\lambda = 2\pi c/\omega = 0.63\ \mu m$ (He-Ne laser), $n_{\parallel} - n_{\perp} \sim 0.1-0.2$, $\delta\vartheta \sim 1$ rad and $L \sim 1$ mm the phase shift of the extraordinary wave

$$\varphi_e(x, y) = \frac{\omega}{c}\int_0^L n_e(\vartheta(x, y, z))dz \qquad (2)$$

has inhomogeneities about $\delta\varphi_e \sim 2\pi(n_{\parallel} - n_{\perp})L/\lambda \sim 10^3$ rad, and for transverse dimension ~ 1 mm of these inhomogeneities the deflection angle is large, $\alpha \sim \delta\varphi_e\,(\lambda/2\pi\Delta x) \sim 0.1$ rad. Therefore the image carried by e-wave is severely distorted.

The main idea of the present work is based on the fact that the refractive index of ordinary wave $n_0 = n_{\perp}$ does not depend on ϑ at all. Therefore the adiabatic propagation of the o-wave in inhomogeneous NLC is not accompanied by phase distortions, $\varphi_0(x, y, z) = \omega z n_{\perp}/c$.

Section 2 of the paper deals with the calculation of the correction to the phase velocity of o-wave due to twist of the NLC and due to externally applied magnetic field. The latter allows in principle to measure the number of half-turns of director. Section 3 describes the results of experimental observation of high transparency of NLC for o-wave and of the pecularities of the light passed near the disclination.

2. CORRECTIONS TO THE PHASE OF ADIABATICALLY PROPAGATING *o*-WAVE

Maxwell equations for monochromatic radiation may be written as follows

$$\text{rot rot}(\hat{\varepsilon}^{-1}\mathbf{D}) - \omega^2/c^2\mathbf{D} = 0 \qquad (3)$$

where $0.5\ \mathbf{D}e^{-i\omega t} + c.\ c$ is the electrical displacement vector, $\hat{\varepsilon}$ is the dielectric susceptibility tensor, and $\hat{\varepsilon}\hat{\varepsilon}^{-1} = 1$. For NLC with the local orientation of the director $\mathbf{n}(\mathbf{r})$ ($|\mathbf{n}| = 1$) the dielectric susceptibility at optical frequency may be taken as

$$\varepsilon_{ij} = \varepsilon_{\perp}\delta_{ij} + (\varepsilon_{\parallel} - \varepsilon_{\perp})n_i n_j + i\mu_1 e_{ijs}H_s + i\mu_2(e_{ips}n_p n_j + n_i n_p e_{pjs})H_s \qquad (4)$$

Here $\varepsilon_{\perp} = n_{\perp}^2$, $\varepsilon_{\parallel} = n_{\parallel}^2$, $n_{\perp}$ and $n_{\parallel}$ are the corresponding refractive indices, μ_1 and μ_2 are the constants which characterize the Faraday-type influence of static magnetic field on the propagation of light. In particular, if one would take $\varepsilon_{\parallel} = \varepsilon_{\perp} = n^2$, $\mu_2 = 0$, then the Faraday rotation would be equal to

$$\frac{d\alpha}{dz}\left(\frac{\text{rad}}{\text{cm}}\right) = \frac{\omega}{c}\frac{\mu_1 H}{2n} \qquad (5)$$

The field $\mathbf{D}(\mathbf{R})$ corresponding to o-wave propagating in z-direction will be taken in the form

$$\mathbf{D}(\mathbf{R}) = e^{ik_0 z}\{A(\mathbf{R})\mathbf{e}_0(\mathbf{R}) + B(\mathbf{R})\mathbf{e}_e(\mathbf{R}) + C(\mathbf{R})\mathbf{e}_z\} \qquad (6)$$

where $k_0 = \omega n_\perp/c$, $|\mathbf{e}_e| = |\mathbf{e}_0| = 1$, $\mathbf{e}_e \sim \mathbf{n}(\mathbf{R}) - \mathbf{e}_z(\mathbf{n}\mathbf{e}_z)$, $\mathbf{e}_0 = [\mathbf{e}_z \times \mathbf{e}_e]$. For homogeneous medium ($\mathbf{n}(\mathbf{R}) = \mathbf{n}_0$ = const) without magnetic field the solution corresponding to o-wave is A = const, $B = C = 0$. If $\mathbf{n}(\mathbf{R})$ has smooth variations and $\mathbf{H} \neq 0$, then the components B and C are admixed to the wave. In the first nonvanishing approximation B contains the terms $\sim (n_\parallel - n_\perp)^{-1}(\mathrm{d}\mathbf{n}/\mathrm{d}z)$ and $\sim (n_\parallel - n_\perp)^{-1}H$, and C contains $(\mathrm{d}\mathbf{n}/\mathrm{d}\mathbf{R}_\perp)$ and $\mathbf{H}$ without small denominator $(n_\parallel - n_\perp)$, see below. For that reason we will neglect the terms C.

Coupled equations for slowly varying amplitudes $A(\mathbf{R})$ and $B(\mathbf{R})$ may be obtained by substitution of (6) into (3) and omission unessential terms

$$\frac{\partial A}{\partial z} = \left\{\frac{n_\perp^2}{n_e^2(\vartheta)}\frac{\partial \varphi}{\partial z} - \frac{1}{2}\left(\frac{\mu_1}{n_e^2(\vartheta)} + \frac{\mu_2 \sin^2\vartheta}{n_\parallel^2}\right)\frac{\omega}{c}n_\perp H\right\} B(z); \qquad (7)$$

$$\frac{n_\perp^2}{n_e^2(\vartheta)}\frac{\partial B}{\partial z} = -\frac{i\omega n_\perp}{2c}\left(\frac{n_\perp^2}{n_e^2(\vartheta)} - 1\right)B(z)$$

$$- \left\{\frac{\partial \varphi}{\partial z} - \frac{1}{2}\left(\frac{\mu_1}{n_e^2(\vartheta)} + \frac{\mu_2 \sin^2\vartheta}{n_\parallel^2}\right)\frac{\omega}{c}n_\perp H\right\}A(z) \qquad (8)$$

Here we assume that the director is characterized by polar angle $\vartheta(\mathbf{R})$ and azimutal one $\varphi(\mathbf{R})$

$$\mathbf{n}(\mathbf{R}) = \mathbf{e}_z \cos\vartheta + \mathbf{e}_x \sin\vartheta\cos\varphi + \mathbf{e}_y \sin\vartheta\sin\varphi \qquad (9)$$

Since we took the main exponential dependence $\exp(ik_0z)$, these equations are good for the situation when $|B| << A$. In the first approximation B follows quasistatically the original amplitude A. To get it, one should neglect $\partial B/\partial z$ in (8), and then

$$B \approx \frac{ic}{\omega(n_\perp - n_\parallel)}\left\{\frac{\partial\varphi}{\partial z} - \frac{1}{2}\left(\mu_1 + \mu_2\sin^2\vartheta\right)\frac{\omega}{cn}H\right\}A(z) \qquad (10)$$

The dimensionless small parameter which determines the precision of adiabatical following is the ratio of $(\mathrm{d}\varphi/\mathrm{d}z)$ to the mismatch $\omega(n_e(\vartheta) - n_0)/c$ between wave vectors k_e and k_0. The admixture of e-wave qualitatively may be explained in the following way. The o-polarization "would like" to propagate with its original vector $\mathbf{e}_0(z_0)$. However at the crossection $(z_0 + h)$ the new polarization vector $\mathbf{e}_0(z_0 + h) \approx \mathbf{e}_0(z_0 + (h\cdot \mathrm{d}\varphi/\mathrm{d}z)\mathbf{e}_e(z_0)$ contains a portion of extraordinary ort $\mathbf{e}_e$. The effective length at which such transformation may be considered as coherent is the mismatch length $h = c/[\omega(n_0 - n_e)]$ and $h\cdot\mathrm{d}\varphi/\mathrm{d}z$ just gives the correct estimation of admixture B.

It is interesting to note that if the magnetic field is present, than the Faraday effect may produce the necessary rate of rotation "to adjust" the turned o-wave to the new twisted ort $\mathbf{e}_0(z_0 + h)$. That may be considered as "explanation" for the combined contributions due to $d\varphi/dz$ and due to H, which may compensate each other for the proper value of H.

The admixed e-wave produces back influence on the original o-wave, and the result is the additional phase shift for the latter. Namely, substitution of B from (10) to (7) gives

$$\frac{\partial A}{\partial z} = i\delta k_0(z)A \tag{11a}$$

$$\delta k(z) \approx \frac{c}{\omega(n_\perp - n_\parallel)}\left\{\frac{\partial\varphi}{\partial z} - \frac{1}{2}\left(\mu_1 + \mu_2 \sin^2\vartheta\right)\frac{\omega}{cn}H\right\}^2 \tag{11b}$$

Let us consider first the case when $H = 0$. The additional phase shift γ_0 for o-wave is

$$\gamma_0(x, y) = \int_0^L \delta k(x, y, z)\mathrm{d}z + m\pi$$

Here the term $m\pi$ describes possible change of the sign of o-wave, when the angle $\varphi(z)$ gets $m/2$ times whole 360° turns along the ray $x = x_0$, $y = y_0$, $0 < z < L$:

$$m = \frac{1}{\pi}\int \frac{\partial\varphi}{\partial z}\mathrm{d}z \tag{12}$$

The boundary between the regions with different values of m corresponds to the projection of a disclination line from the LC volume to the (x, y) plane.

Suppose there is some local region, at which given value $d\varphi/dz$ is almost constant at a longitudinal dimension l. Then the number of independent contributions to $\gamma_0(x, y)$ is about $N = L/l$, and the overall fluctuation $\delta\gamma_0$ is about

$$\delta\gamma_0 \approx N^{1/2}l|\delta k| \approx \frac{c}{\omega}(Ll)^{1/2}(\mathrm{d}\varphi/\mathrm{d}z)^2/(n_e - n_0) \tag{13}$$

Here are the numerical estimations. Let $l \approx 10^{-2}$ cm, $|d\varphi/dz| \approx 1$ rad/10^{-2} cm $= 10^{+2}$ cm^{-1}, $L \approx 10^{-1}$ cm, $N = 10$, $n_e - n_0 \approx 0.15$. Then $\delta\gamma_0 \approx 6.7\cdot 10^{-3}$ rad, and that means very small distortion of the phase of o-wave. We would like to raise an interesting physical question, whether it is possible to measure the number m of half-rotations of o-polarization along the ray. The introduction of magnetic field was made just with that aim in the mind. Namely, one should measure the difference between phase shifts for $\mathbf{H} = H_0\mathbf{e}_z$ and $\mathbf{H} = -H_0\mathbf{e}_z$,

$$\Delta\gamma = \gamma(x_0, y_0, H_0) - \gamma(x_0, y_0, -H_0) = \frac{2}{n}H_0\int \frac{\mu_1 + \mu_2\sin^2\vartheta}{n_e(\vartheta) - n_\perp}\frac{\partial\varphi}{\partial z}\mathrm{d}z \tag{14}$$

If $\vartheta(z) = \pi/2$, $\varphi = \varphi(z)$, then the phase difference equals to

$$\Delta\gamma \approx 2\pi\frac{(\mu_1 + \mu_2)}{n(n_\parallel - n_\perp)}mH_0 \tag{15}$$

Even in the case of $\vartheta = \vartheta(z)$ the equation (14) shows, that some information about $d\varphi/dz$ is accumulated in $\delta\gamma$.

It is worth mentioning that the phase $\gamma(H_0) - \gamma(-H_0)$ is accumulated during the subsequent passes of the beam back and forth through the cell, just like the time-odd Faraday effect. The discussion of the effects of such a type was previously given in References 1 and 2.

The numerical estimations are not very optimistic for Faraday measurements. For $\vartheta(z) = \pi/2$, $d\varphi/dz = \pi/L$ (i.e. for $m = 1$), $\mu_1 \approx 10^{-10}$ rad/oersted (corresponds to Verdet constant $r \approx 3.25 \cdot 10^{-6}$ rad/cm · oersted $\approx 10^{-2}$ min/cm · oersted at λ = 6328Å) one gets $\Delta\gamma \approx 2 \cdot 10^{-9}$ rad/oersted. It means that in the external magnetic field $H \approx 10^4$ oersted o-wave, transmitted through different regions of NLC with disclinations obtains phase difference $\sim 2 \cdot 10^{-5}$ rad for one full circulation of polarization vector.

The main conclusions of our theoretical consideration are: 1) there is possibility to transmit o-image through a relatively thick NLC cell; 2) there will be π-jumps in the phase of the transmitted wave; 3) application of longitudinal magnetic field permits in principle to reveal the number of π-rotations of polarization.

3. EXPERIMENTAL RESULTS

The experimental setup is shown at the Figure 1. Light from a slide-projector or from a laser was transmitted through a polarizer, which allowed to change the input polarization. After that the beam hit the cell with liquid crystal. For the case of a slide-projector the image of a slide was focused onto the screen or to a photofilm. For the case of a laser beam the latter was made either parallel or slightly divergent one by the use of a telescope or a lens. The cell was made of two 4 mm thick glass plates with teflon gasket from 0.4 mm to 5 mm thick between them. We used nematic liquid crystal 5CB. The planar orientation of LC at the walls was achieved by standard technique of covering the glass by polyvinyl and mechanical polishing in necessary direction. However, the orientation of the di-

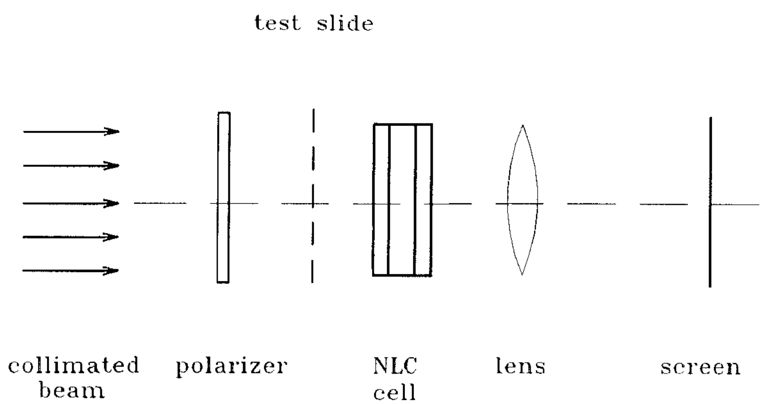


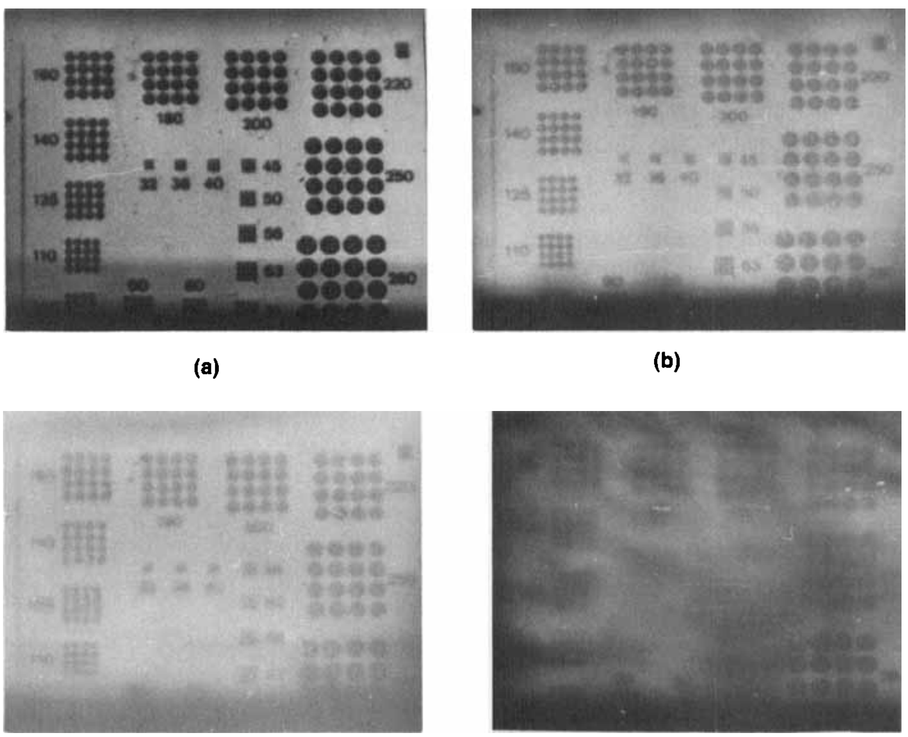
FIGURE 1 The scheme of experiment on observation of distortionless transmittance of NLC cell for ordinary wave.

rector was not regular inside the volume of the cell, especially for a cell with thickness more than 1 mm.

Without polarizers the cell looked very opaque, and no picture could be seen through it by naked eye. However, when the polarizer was mounted in such a way that either input or output beam (or both of them) corresponded to the ordinary wave only, the image was transmitted through the cell without visual distortions. We used also a test slide mounted in a slide-projector. The image of the slide was transmitted through the cell and focused to the screen or to a photofilm. Corresponding pictures of the original slide, of its image by *o*-wave and by *e*-wave are shown at the Figure 2.

The transverse dimensions of the cell were about 20 × 30 mm, and almost all the cell was illuminated in those experiments. One can see that *o*-wave allowed to transmit the image almost without distortion even for the cell 5 mm thick. At the same time, *e*-wave was strongly distorted already by 1 mm thick cell, see Figure 2. No image at all could be seen at *e*-wave through cells with thickness $d > 3$ mm.

The other set of experiments was done with the use of He-Ne laser beam. A collimated beam with the diameter about 2 mm was directed to the cell either as *o*-wave or as *e*-wave. The light transmitted through the cell consisted of two different



(a) (b) (c) (d)

FIGURE 2 The original test slide (a). The images of the test slides produced by *o*-wave transmitted through the NLC cells of 1 mm (b) and 5 mm (c) thick. The same for *e*-wave, 1 mm thick cell (d).

parts. One of them corresponded to the diffuse wide-angle scattering of incident light. That was apparently the scattering of light by thermodynamically equilibrium fluctuations of orientation of the director (so-called molecular scattering). For $d \approx 3$ mm that wide-angle light was almost totally unpolarized, independently of the type of incident wave, *o* or *e*. The other part of transmitted light corresponded to the image and just to the output of the cell it was concentrated more or less inside the cross-section of incident beam. That image-carrying part of the beam was considerably attenuated by molecular scattering (see below). However, it was a pleasant surprise for us that the degree of polarization of that part of the transmitted beam turned out to be very high (>90%) for either *e*- or *o*-type of incident wave. That means that the idea of adiabatical following of polarization of a propagating wave is applicable to our experiment very well.

The *e*-wave was strongly distorted, apparently due to static inhomogeneities of the director inside the cell. The *o*-wave was transmitted almost without distortions. We have measured the fraction of light power, transmitted in *o*-wave in a highly directed beam. That was done with the use of a calorimeter and an aperture (2 mm diameter) before it, mounted at a distance $L \sim 10$ cm from the cell. The transmission was $T = 0.57$ for $d = 0.6$ mm, $T = 0.39$ for $d = 1$ mm and $T = 0.08$ for $d = 3$ mm. If we take into account the Fresnel losses at the input and output glass surface, then the corresponding values are $T_{0.6} = 0.62$, $T_1 = 0.42$ and $T_3 = 0.09$. Figure 3 shows the dependence of ln T on thickness d. The estimation of extinction coefficient R (for intensity) ≈ 8 cm^{-1} more or less coincides with theory, see References 3 and 4.

We should like to note also, that the use of a small permanent magnet made of Co-Sm alloy allowed us to make strong inhomogeneties inside the cell, so that it served as astigmatic lens for *e*-wave. The focal lengths of that lens were about $F \sim (5 \div 15)$ cm for $d \sim 3$ mm. The values of F depended on the time of "exposure" by magnetic field, and the lens were relaxing during 1 to 5 minutes after the field was removed. All these operations did not influence the propagation of *o*-wave.

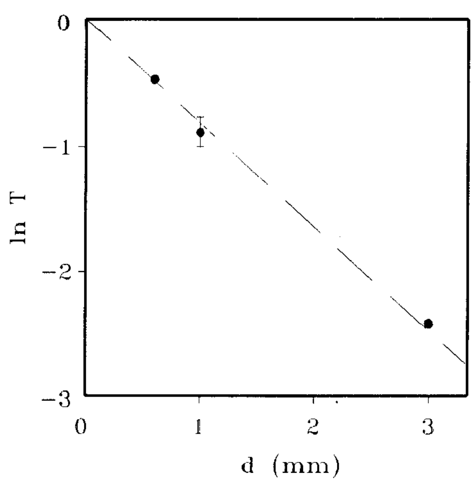


FIGURE 3 The dependence of NLC-cell transmittance $T = I(d)/I(0)$ on the cell thickness for *o*-wave.

Illumination of the cell by a wide laser beam allowed us to see disclinations. There were a lot of them in the field of view just after filling the cell by NLC, and after a day or two their number was much smaller. For a cell with $d = 3$ mm the transverse distance between dislocations was typically about 1 cm, and the shape of a disclination was irregular. Figure 4a shows the glinting of a disclination; that photographs was made for *o*-illumination and *o*-registration at a small angle (~10°) to the incident beam. It should be noted that no glint could be seen for $o \rightarrow e$ registration.

Figure 4b shows the picture of the transmitted light for plane *o*-wave illumination at a distance 200 mm. Theory predicts the phase shift 180° for the *o*-wave transmitted at different sides of disclination.

We have verified that fact by three different methods—two linear optical and one nonlinear optical. First, we have compared the theoretical expression

$$E(z, x) = E_0 e^{ikz} \sqrt{2/\pi i} \int_0^t e^{i\tau^2} d\tau; \qquad t = x\sqrt{k/2z}$$

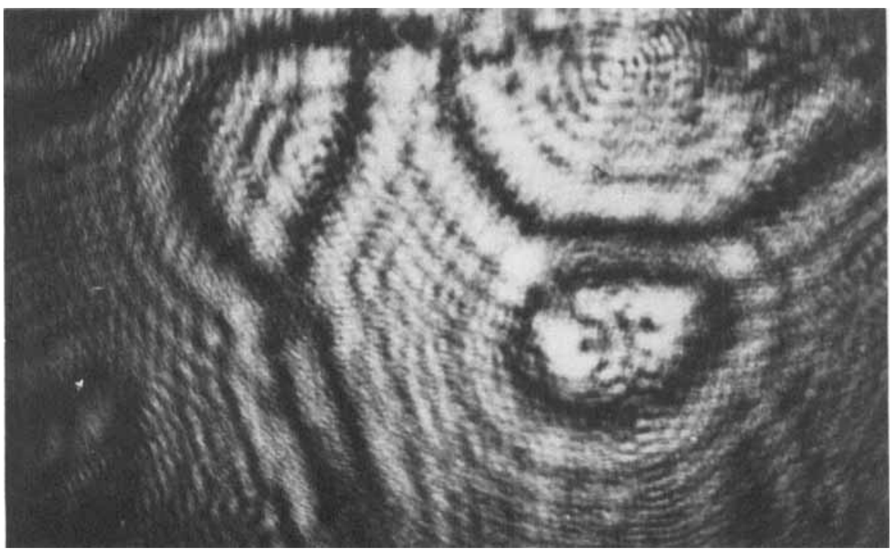
for the Fresnel diffraction of the field $E(z = 0, x) = E_0 x/|x|$ with wave length $\lambda = 2\pi/k$ at a distance z with the experimental distribution of the intensity around disclination. The correspondence was quite reasonable. Secondly, we used the interferometric scheme to register the phase of the *o*-wave just at the output of the cell. The interferogram Figure 5 shows very clearly the 180° shift between the adjacent domains of the wave front.

In the third experiment we used second harmonic (SH, $\lambda = 0.53$ μm) generation by the $\lambda = 1.06$ μm radiation of pulsed Nd-YAG laser. We took two identical KTP crystals with the NLC cell between them. (We are grateful to Yu. E. Kapitzky, A. N. Chudinov and A. Schulginov for the suggestion to use that set-up which was developed by them for other purposes). The amplitude of the output SH was the coherent sum of amplitudes produced in the first and second crystals. We could change the relative phase φ of these contributions by longitudinal shift of second crystal: the dispersion of refractive index of air produced phase shift $\varphi_{2\omega} - 2\varphi_\omega = 2\omega \cdot L(n(2\omega) - n(\omega))/c$ at a distance L, and the corresponding interference dependence $I_{2\omega}(L) \sim A + B \cdot \cos(\varphi_0 + (\varphi_{2\omega} - 2\varphi_\omega))$ was registered. We measured the change of the value φ_0, when both beams ω and 2ω were shifted from one transverse position to the other to jump over the dislocation. The experimentally measured value was $\Delta\varphi_0 = 2.8$ rad $\pm$ 0.4 rad. Theory predicts $\Delta\varphi_0 = \pi$ rad, since the transmission of *o*-wave at different sides from disclination gives the change of sign for $\mathbf{E}_\omega$ and $\mathbf{E}_{2\omega}$, but does not change the sign of $\mathbf{E}_\omega$ and $\mathbf{E}_\omega$, which is the source of SH in the second crystal.

4. DISCUSSION

Here we would like to discuss the difference between the undistorted transmitted *o*-wave-image and the multiply scattered wave with the same polarization which goes inside the solid angle occupied by the image. The difference is that the un-

(a)



(b)

FIGURE 4 The picture of a disclination glinting in 1 mm thick NLC-cell for o-wave illumination (a). Diffraction pattern on disclinations in NLC-cell (1 mm thick, o-wave) at the distance 200 mm from the cell (b).

distorted image, even being very strongly attenuated by the scattering, keeps its temporal coherence with the incident illuminating beam, while the molecular scattering entirely destroys that coherence having spectral deviations $\Omega/2\pi \sim (10 \div 100)$ Hz. This distinction allows us to register the undistorted o-wave by amplifying it in a photorefractive crystal pumped by the same illuminating laser beam, since

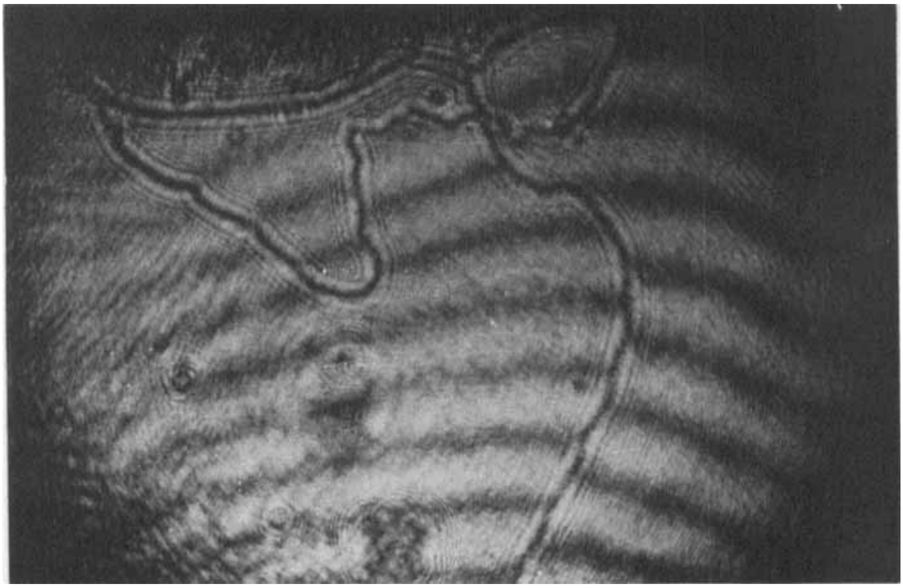


FIGURE 5 The interference between the reference wave and *o*-wave transmitted through 1 mm thick NLC-cell with disclinations. The shift of interference fringes on disclination line corresponds to 180°-phase difference.

the response time of a photorefractive crystal $\tau \sim (1 \div 10)$ sec will not allow to amplify the molecularly scattered waves with such large frequency shift.

We hope that the effect of undistorted propagation of *o*-wave through the crystal will find some applications both in science and in technology.

Acknowledgment

Authors are grateful to B. Ya. Meteliza for assistance in experiments.

References

1. N. B. Baranova and B. Ya. Zel'dovich, *Mol. Physics*, **38**(4), 1085–1098 (1979).
2. O. S. Yerician, *Usp. Fiz. Nauk*, **138**, 645–674 (Soviet Phys. Usp., 1982).
3. P. G. DeGennes, *Compt. Rend.*, **266**, 15 (1968).
4. D. Langevin and M. Bouchiat, *Journ. Phys.* (Fr.), **36**, Suppl. C1, 197 (1975).